Substitution and income effect

A consumer's preferences over goods x and y are represented by the utility function $u(x, y) = x^{1/2} + y^{1/2}$.

1. Derive the general formula for the demands of x and y as functions of the prices p_x and p_y and the income I, and calculate the optimal consumption bundle if $I = 10$ and the prices of the goods are $p_x = 1$ and $p_y = 1$.
2. Suppose the State imposes a tax on good y and as a result p_y increases to 2 while p_x and I remain constant. Calculate the optimal bundle after the tax. Calculate the income and substitution effects on the demand for good y from the increase in p_y.

Solution

1. $$MRS = \frac{(1/2)x^{-1/2}}{(1/2)y^{-1/2}} = \frac{y^{1/2}}{x^{1/2}} = \sqrt{\frac{y}{x}}$$

$$\sqrt{\frac{y}{x}} = \frac{p_x}{p_y}$$

$$y/x = \frac{p_x^2}{p_y^2}$$

$$y = x\frac{p_x^2}{p_y^2}$$

   Insert into the budget constraint

$$p_x x + p_y x\frac{p_x^2}{p_y^2} = I.$$

   Solving the equation:

$$x(p_x + \frac{p_x^2}{p_y}) = I$$

$$x(\frac{p_x p_y + p_x^2}{p_y}) = I$$

$$x = \frac{p_y I}{p_x p_y + p_x^2}$$

$$x(p_x, p_y, I) = \frac{I p_y}{p_x(p_x + p_y)}$$

   Insert into the equation for y

$$y = \frac{I p_y}{p_x(p_x + p_y)}\frac{p_x^2}{p_y^2}$$

$$y(p_x, p_y, I) = \frac{I p_x}{p_y(p_x + p_y)}$$

   Therefore, $x(1, 1, 10) = 5 = y(1, 1, 10)$.

2. Using the demand functions calculated in the previous part, we get:

$$x(1, 2, 10) = \frac{20}{3}$$

$$y(1, 2, 10) = \frac{10}{6}.$$

   Therefore, the total effect on the demand for y is

$$TE = y(1, 2, 10) - y(1, 1, 10) = -\frac{20}{6} = -3.3.$$

To calculate the substitution effect, we solve

$$\left(\frac{2I}{3}\right)^{\frac{1}{2}} + \left(\frac{I}{6}\right)^{\frac{1}{2}} = (5)^{\frac{1}{2}} + (5)^{\frac{1}{2}}$$

$$\frac{\sqrt{4I} + \sqrt{I}}{\sqrt{6}} = 4.4721$$

$$3\sqrt{I} = \sqrt{6}4.4721$$

$$I = 13.33$$

With this, we obtain the demand for y:

$$y(1, 2, 13.33) = \frac{13.33}{6} = 2.22$$

With this, we find the substitution effect:

$$SE = y(1, 2, 13.33) - y(1, 1, 10) = 2.22 - 5 = -2.78$$

The income effect is

$$IE = TE - SE = -0.53.$$